8/29 Recitation

In this recitation, you will use the commands found in the **Julia2Matrices.ipynb** and **Julia3Proofs.ipynb** notebooks; these can be found on canvas. Open a new Jupyter notebook and label it something like **Recitation8.29** (or add to the previous Recitation notebook, if you'd prefer to have one large recitation notebook). Do all of your Julia calculations today in that notebook; be sure to save your work so you can reference it later!

1. Show that the following statements are false by finding a counterexample.
   (a) If AB and BA are defined, then A and B are square matrices.

   (b) If $AB = B$ for some matrix B, then $A = I$

2. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 5 & -1 \\ -2 & 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 1 & -3 \\ 0 & 3 \end{bmatrix}$. Compute AB and BA, if they exist. Verify your computations using Julia.

3. Prove that if A is $m \times n$ and $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$, then $A(\mathbf{v}+\mathbf{w}) = A\mathbf{v} + A\mathbf{w}$. Hint: Use the linear combination definition of multiplication and the vector property: $(a+b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}$.

4. Use Julia to symbolically prove the associativity property $A(BC) = (AB)C$ for some symbolic matrices A, B, and C of any (appropriate) size you choose.

5. Consider the function $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + y \\ x - y \\ 2x \end{bmatrix}.$$

   $\mathbb{R}^2$ is called the **domain** of T and $\mathbb{R}^3$ is called the **codomain** of T.

   - Evaluate $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$.

   - Sketch the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ in the domain of T and the vector $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$ in the codomain of T.

Looking Ahead

6. Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x - y \\ x + y \end{bmatrix}$. Find a vector $\begin{bmatrix} a \\ b \end{bmatrix}$ in the domain satisfying $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ by reinterpreting this equation as a system of equations and solving that system.